

Monster Questions – Set 1

QUESTION 1

The diagram shows a circular pond, of radius r metres, surrounded by a circular path. The circular path has a constant width of 1.5 metres.

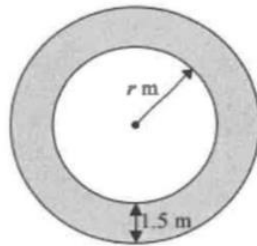


Diagram NOT
accurately drawn

$$\pi(r+1.5)^2 - \pi r^2 = \frac{\pi r^2}{10}$$

The area of the path is $\frac{1}{10}$ the area of the pond.

(a) Show that $2r^2 - 60r - 45 = 0$

$$\pi[(r+1.5)^2 - r^2] = \frac{\pi r^2}{10}$$

$$\pi[r^2 + 3r + 2.25 - r^2] = \frac{\pi r^2}{10}$$

$$3r + 2.25 = \frac{r^2}{10}$$

$$30r + 22.5 = r^2$$

$$0 = r^2 - 30r - 22.5$$

$$2r^2 - 60r - 45 = 0$$

(b) Calculate the area of the pond.

Show your working clearly.

Give your answer correct to 3 significant figures.

$$2r^2 - 60r - 45 = 0$$

$$r = \frac{60 \pm \sqrt{(60^2 - 4 \times 2 \times -45)}}{4}$$

$$= 30.732$$

$$A = \pi \times 30.732^2$$

$$= 2967.000$$

$$= 2970 \text{ m}^2$$

QUESTION 2

Correct to 2 significant figures, $a = 58$, $b = 28$ and $c = 18$

Calculate the upper bound for the value of $\frac{a}{b-c}$

Show your working clearly.

$$\begin{array}{rcl}
 a & 57.5 & 58.5 \\
 b & 27.5 & 28.5 \\
 c & 17.5 & 18.5
 \end{array}
 \quad = \quad \frac{58.5}{27.5 - 18.5} = 6.5$$

QUESTION 3

Two bags contain discs.

Bag A contains 12 discs.

5 of the discs are red, 6 are blue and 1 is white.

Bag B contains 25 discs.

n of the discs are red and the rest are blue.

James takes at random a disc from Bag A.

Lucy takes at random a disc from Bag B.

Given that the probability that James and Lucy both take a red disc is $\frac{2}{15}$

(i) find the value of n , the number of red discs in Bag B.

$$\frac{5}{12} \times \frac{n}{25} = \frac{2}{15}$$

$$\frac{n}{60} = \frac{2}{15} \quad n = 8$$

$$n = 8$$

(ii) Hence calculate the probability that James and Lucy take discs of different colours.

$$P(\text{same}) = P(RR) + P(BB)$$

$$= \frac{2}{15} + \left[\frac{6}{12} \times \frac{17}{25} \right]$$

$$= \frac{2}{15} + \frac{17}{50}$$

$$= \frac{71}{150}$$

$$\begin{aligned}
 P(\text{diff}) &= 1 - \frac{71}{150} \\
 &= \frac{79}{150}
 \end{aligned}$$

QUESTION 4

▭ $ABCD$ is a kite.

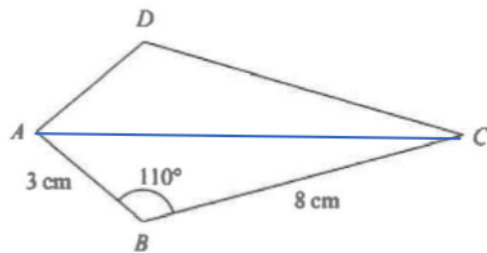


Diagram NOT
accurately drawn

$AB = 3 \text{ cm}$
 $BC = 8 \text{ cm}$
 Angle $ABC = 110^\circ$

Calculate the area of the kite $ABCD$.
 Give your answer correct to 3 significant figures.

$$A = \frac{1}{2} ab \sin C$$

$$2 \times \frac{1}{2} \times 3 \times 8 \times \sin 110$$

$$A (\text{kite}) = 22.6 \text{ cm}^2$$

QUESTION 5

Solve $x^2 + y^2 = 20$
 $y = 10 - 2x$ ←

Show clear algebraic working.

$$x^2 + (10 - 2x)^2 = 20$$

$$x^2 + 100 - 40x + 4x^2 = 20$$

$$5x^2 - 40x + 80 = 0$$

$$x^2 - 8x + 16 = 0$$

$$(x - 4)(x - 4) = 0$$

$$x = 4$$

$$\begin{aligned} y &= 10 - 2x \\ &= 10 - 8 \\ &= 2 \end{aligned}$$

$$(4, 2)$$