

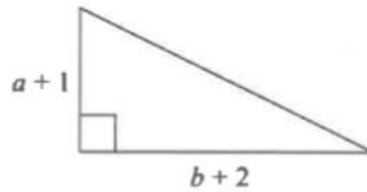
Monster Questions – Set 2

Question 1

The diagram shows two triangles, A and B.



Triangle A



Triangle B

Diagram NOT accurately drawn

The area of triangle B is 3 times the area of triangle A.

Given that $b > 4$, find an expression for a in terms of b .

$$\begin{aligned} A &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} ab \sin 150 \\ &= \frac{ab}{4} \end{aligned}$$

$$\begin{aligned} A &= \frac{(b+2)(a+1)}{2} \\ &= \frac{ab + 2a + b + 2}{2} \end{aligned}$$

$$\frac{ab + 2a + b + 2}{2} = 3 \times \frac{ab}{4}$$

$$2ab + 4a + 2b + 4 = 3ab$$

$$-ab + 4a = -2b - 4$$

$$4a - ab = -2b - 4$$

$$a(4 - b) = -2b - 4$$

$$a = \frac{-2b - 4}{4 - b}$$

Question 2

n is a positive integer.

(a) Explain why $2n + 1$ is an odd number for all values of n .

$$2n = \text{even (multiple of 2)}$$

$$2n + 1 = \text{odd}$$

(1)

(b) Show, using algebra, that the sum of any 4 consecutive odd numbers is always a multiple of 8

$$\begin{aligned} & 2n + 1 + 2n + 3 + 2n + 5 + 2n + 7 \\ &= 8n + 16 \\ &= 8(n + 2) \end{aligned}$$

Question 3

(a) Show that $(5 - \sqrt{8})(7 + \sqrt{2}) = 31 - 9\sqrt{2}$

Show each stage of your working.

$$\begin{aligned} & (5 - \sqrt{8})(7 + \sqrt{2}) \\ &= 35 + 5\sqrt{2} - 7\sqrt{8} - \sqrt{8}\sqrt{2} \\ &= 35 + 5\sqrt{2} - 7\sqrt{4 \times 2} - \sqrt{16} \\ &= 35 + 5\sqrt{2} - 14\sqrt{2} - 4 \\ &= 31 - 9\sqrt{2} \end{aligned}$$

Given that c is a prime number,

(b) rationalise the denominator of $\frac{3c - \sqrt{c}}{\sqrt{c}}$

Simplify your answer.

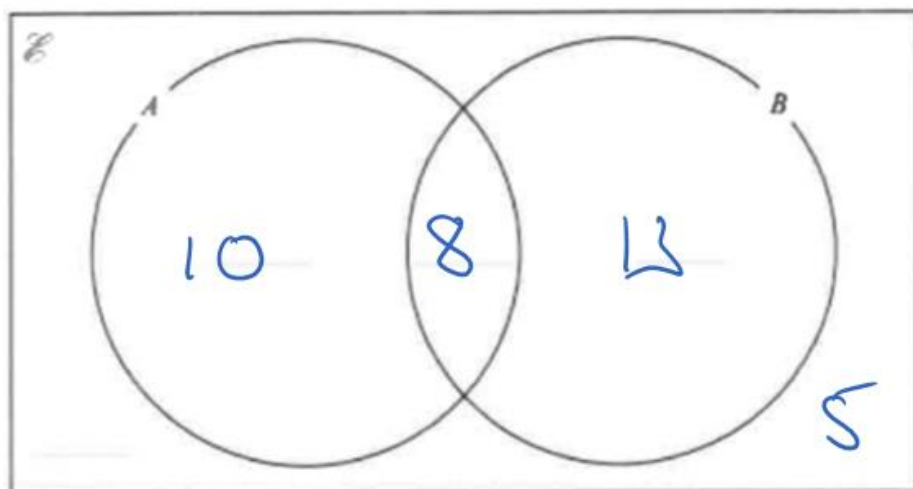
$$\begin{aligned} \frac{3c - \sqrt{c}}{\sqrt{c}} \times \frac{\sqrt{c}}{\sqrt{c}} &= \frac{3c\sqrt{c} - c}{c} \\ &= \frac{c(3\sqrt{c} - 1)}{c} \\ &= 3\sqrt{c} - 1 \end{aligned}$$

Question 4

$\mathcal{U}$ A and B are two sets.

$$\begin{aligned} n(\mathcal{U}) &= 36 \\ n(B) &= 21 \\ \checkmark \quad n(A \cap B) &= 8 \\ n(A') &= 18 \end{aligned}$$

(a) Complete the Venn diagram to show the **number of elements** in each region of the Venn diagram.



(3)

(b) Find $n(A \cup B)$

$$10 + 8 + 13$$

$$31$$

(1)

(c) Find $n(A \cap B')$

A and B'

$$10$$

Question 5

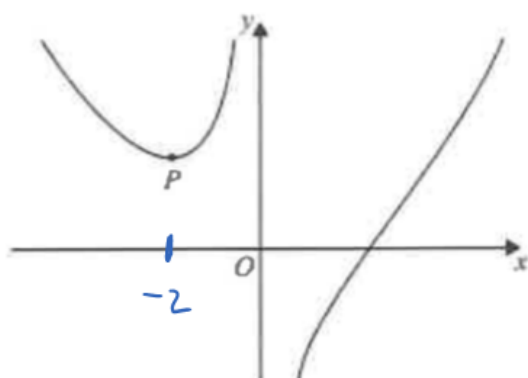
$$y = x^2 - \frac{16}{x}$$

(a) Find $\frac{dy}{dx}$

$$y = x^2 - 16x^{-1}$$

$$\frac{dy}{dx} = 2x + 16x^{-2}$$

$$\frac{dy}{dx} = 2x + \frac{16}{x^2} \quad (3)$$



The graph shows part of the curve with equation $y = x^2 - \frac{16}{x}$

The point P is the turning point of the curve.

(b) Work out the coordinates of P .

$$2x + \frac{16}{x^2} = 0$$

$$2x^3 + 16 = 0$$

$$2x^3 = -16$$

$$x^3 = -8$$

$$x = -2$$

$$y = x^2 - \frac{16}{x}$$

$$x = -2$$

$$\begin{aligned} y &= 4 - \frac{16}{-2} \\ &= 4 + 8 \\ &= 12 \end{aligned}$$

$$P(-2, 12)$$