

Monster Questions – Set 4

Question 1

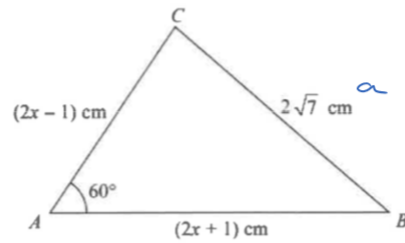


Diagram NOT
accurately drawn

The diagram shows a triangle ABC .

$AB = (2x + 1)$ cm, $AC = (2x - 1)$ cm and $BC = 2\sqrt{7}$ cm.

Angle $BAC = 60^\circ$

Work out the value of x .

Show clear algebraic working.

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$(2\sqrt{7})^2 = (2x-1)^2 + (2x+1)^2 - 2(2x-1)(2x+1) \cos 60$$

$$28 = 4x^2 - 4x + 1 + 4x^2 + 4x + 1 - [4x^2 - 1]$$

$$28 = 4x^2 + 2$$

$$4x^2 = 25$$

$$x^2 = \frac{25}{4}$$

$$x = \frac{5}{2}$$

Question 2

$PQRS$ and $PLMN$ are similar quadrilaterals.

$PN = 12$ cm, $NS = 8$ cm, $PL = 9$ cm and $RS = 13.5$ cm.

LM is parallel to QR and MN is parallel to RS .

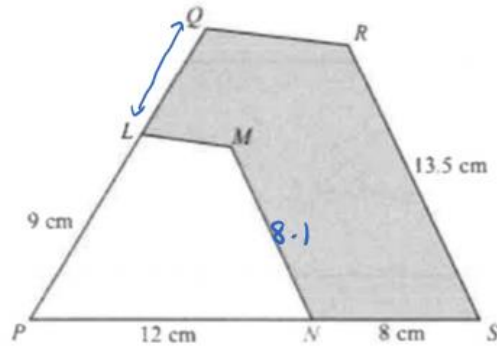


Diagram NOT
accurately drawn

(a) Work out the length of MN .

$$\begin{array}{l} PS \quad 20 \\ PN \quad 12 \end{array} \quad \times \frac{5}{3} \quad x = 13.5 \div \frac{3}{5} = 8.1$$

_____ cm
(2)

(b) Work out the length of LQ .

$$\begin{aligned} PQ &= \frac{5}{3} \times 9 = 15 \\ LQ &= 15 - 9 = 6 \end{aligned}$$

_____ cm
(2)

The area of $PLMN$ is A cm²
The area of $PQRS$ is kA cm²

(c) Find the value of k .

$$\xrightarrow{\times \frac{5}{3}} \left(\frac{5}{3}\right)^2 = \frac{25}{9}$$

The area of the shaded region is 105.6 cm²

(d) Work out the value of A .

$$\begin{aligned} \frac{25}{9}A - A &= 105.6 \\ \frac{16}{9}A &= 105.6 \Rightarrow A = 59.6 \text{ cm}^2 \end{aligned}$$

Question 3

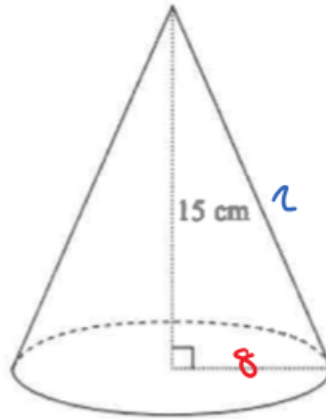


Diagram NOT
accurately drawn

A solid cone has a height of 15 cm.
The volume of the cone is $320\pi \text{ cm}^3$

$$V = \frac{1}{3}\pi r^2 h$$

Work out the curved surface area of the cone.
Give your answer correct to 3 significant figures.

$$CSA = \pi r l$$

$$\frac{1}{3}\pi r^2 h = 320\pi$$

$$l = \sqrt{15^2 + 8^2}$$

$$l = 17$$

$$\frac{1}{3}r^2 \times 15 = 320$$

$$CSA = \pi \times 8 \times 17$$

$$5r^2 = 320$$

$$= 427 \text{ cm}^2$$

$$r^2 = 64$$

$$r = 8$$

Question 4

$$f: x \mapsto 2x^2 + 1 \quad g: x \mapsto \frac{2x}{x-1} \quad \text{where } x \neq 1$$

- (a) Express the composite function gf in the form $gf: x \mapsto \dots$.
Give your answer as simply as possible.

$$\begin{aligned} gf(x) &= f(x) = 2x^2 + 1 \\ gf(x) &= \frac{2(2x^2 + 1)}{2x^2 + 1 - 1} \\ &= \frac{2(2x^2 + 1)}{2x^2} \\ &= \frac{2x^2 + 1}{x^2} = 2 + \frac{1}{x^2} \end{aligned}$$

$$gf: x \mapsto \frac{2 + \frac{1}{x^2}}{(2)}$$

- (b) Express the inverse function g^{-1} in the form $g^{-1}: x \mapsto \dots$

$$g(x) = \frac{2x}{x-1}$$

$$y = \frac{2x}{x-1}$$

$$y(x-1) = 2x$$

$$yx - y = 2x$$

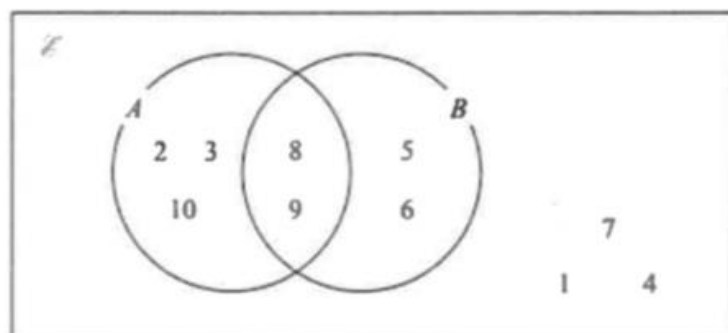
$$yx - 2x = y$$

$$x(y-2) = y$$

$$x = \frac{y}{y-2}$$

$$g^{-1}: x \rightarrow \frac{x}{x-2}$$

Question 5



The Venn diagram shows all of the elements in sets A , B and U .

(a) Write down the elements in A'

5, 6, 1, 7, 4
(1)

(b) Find $n(A \cap B)'$

8
(1)

(c) Find the elements in $(A \cap B) \cup (A \cup B)'$

8, 9 or 7, 1, 4

8, 9, 7, 1, 4
(1)

$$A \cap C = \emptyset$$

$$B \cup C = \{5, 6, 7, 8, 9\}$$

$$n(C) = 3$$

(d) Write down the elements in C .

7, 5, 6